UK SENIOR MATHEMATICAL CHALLENGE

Tuesday 7 November 2017

Organised by the **United Kingdom Mathematics Trust**

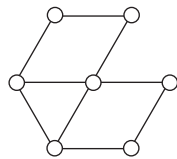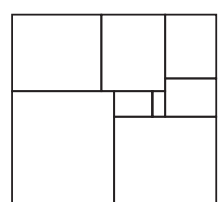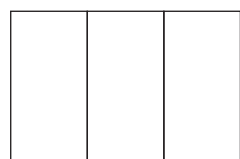and supported by

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RULES AND GUIDELINES (to be read before starting)

1. Do not open the question paper until the invigilator tells you to do so.

2. Time allowed: **90 minutes**.
    No answers or personal details may be entered on the Answer Sheet after the 90 minutes are over.

3. The use of rough paper is allowed.
    Calculators, measuring instruments and squared paper are forbidden.

4. Candidates must be full-time students at secondary school or FE college, and must be in Year 13 or below (England & Wales); S6 or below (Scotland); Year 14 or below (Northern Ireland).

5. **Use B or HB pencil only**. Mark *at most one* of the options A, B, C, D, E on the Answer Sheet for each question. Do not mark more than one option.

6. **Scoring rules**: all candidates start out with 25 marks;

    > 0 marks are awarded for each question left unanswered;

    > 4 marks are awarded for each correct answer;

    > **1 mark is deducted** for each incorrect answer.

7. **Guessing**: Remember that there is a penalty for incorrect answers. Note also that later questions are deliberately intended to be harder than earlier questions. You are thus advised to concentrate first on solving as many as possible of the first 15-20 questions. Only then should you try later questions.

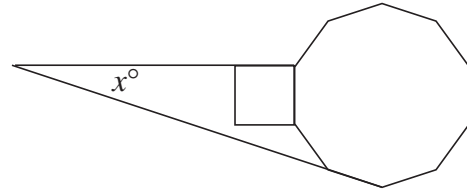**The United Kingdom Mathematics Trust is a Registered Charity.**

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1. One of the following numbers is prime. Which is it?

    A 2017 − 2   B 2017 − 1   C 2017   D 2017 + 1   E 2017 + 2

2. Last year, an earthworm from Wigan named Dave wriggled into the record books as the largest found in the UK. Dave was 40 cm long and had a mass of 26 g. What was Dave's mass per unit length?

    A 0.6 g/cm   B 0.65 g/cm   C 0.75 g/cm   D 1.6 g/cm   E 1.75 g/cm

3. The five integers 2, 5, 6, 9, 14 are arranged into a different order. In the new arrangement, the sum of the first three integers is equal to the sum of the last three integers. What is the middle number in the new arrangement?

    A 2   B 5   C 6   D 9   E 14

4. Which of the following is equal to $2017 - \dfrac{1}{2017}$?

    A $\dfrac{2017^2}{2016}$ B $\dfrac{2016}{2017}$ C $\dfrac{2018}{2017}$ D $\dfrac{4059}{2017}$ E $\dfrac{2018 \times 2016}{2017}$

5. One light-year is nearly 6×10^{12} miles. In 2016, the Hubble Space Telescope set a new cosmic record, observing a galaxy 13.4 thousand million light-years away. Roughly how many miles is that?

    A 8×10^{20} B 8×10^{21} C 8×10^{22} D 8×10^{23} E 8×10^{24}

6. The circles in the diagram are to be coloured so that any two circles connected by a line segment have different colours.

    What is the smallest number of colours required?

    A 2   B 3   C 4   D 5   E 6

7. The positive integer k satisfies the equation $\sqrt{2} + \sqrt{8} + \sqrt{18} = \sqrt{k}$. What is the value of k?

    A 28   B 36   C 72   D 128   E 288

8. When evaluated, which of the following is not an integer?

    A 1^{-1} B $4^{-\frac{1}{2}}$ C 6^0 D $8^{\frac{2}{3}}$ E $16^{\frac{3}{4}}$

9. The diagram shows an $n \times (n + 1)$ rectangle tiled with $k \times (k + 1)$ rectangles, where n and k are integers and k takes each value from 1 to 8 inclusive.

    What is the value of n?

    A 16   B 15   C 14   D 13   E 12

10. A rectangle is divided into three smaller congruent rectangles as shown.

    Each smaller rectangle is similar to the large rectangle.

    In each of these four rectangles, what is the ratio of the length of a longer side to that of a shorter side?

    A $2\sqrt{3} : 1$ B $3 : 1$ C $2 : 1$ D $\sqrt{3} : 1$ E $\sqrt{2} : 1$

11. The teenagers Sam and Jo notice the following facts about their ages:
    The difference between the squares of their ages is four times the sum of their ages.
    The sum of their ages is eight times the difference between their ages.
    What is the age of the older of the two?

    A 15   B 16   C 17   D 18   E 19

12. The diagram shows a square and a
    regular decagon that share an edge. One
    side of the square is extended to meet an
    extended side of the decagon.
    What is the value of x?

   A 15   B 18   C 21   D 24   E 27

13. Isobel: "Josh is innocent"   Genotan: "Tegan is guilty"
    Josh: "Genotan is guilty"   Tegan: "Isobel is innocent"
    Only the guilty person is lying; all the others are telling the truth. Who is guilty?

    A Isobel   B Josh   C Genotan D Tegan   E More information required

14. In the diagram, all the angles marked • are equal in size
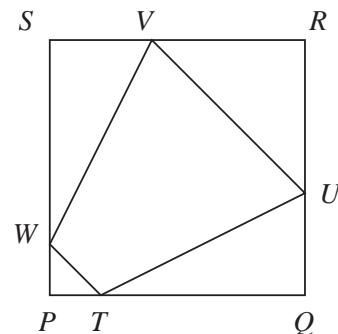   to the angle marked $x°$.
    What is the value of x?

    A 100   B 105   C 110   D 115   E 120
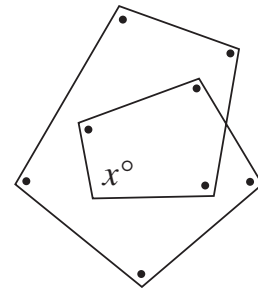
15. The diagram shows a square $PQRS$. Points T, U, V
    and W lie on the edges of the square as shown, such
    that $PT = 1, QU = 2, RV = 3$ and $SW = 4$.
    The area of $TUVW$ is half that of $PQRS$. What is
    the length of PQ?
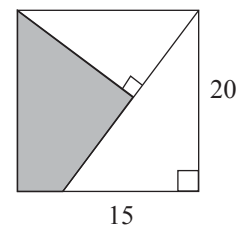
    A 5   B 6   C 7   D 8   E 9

16. The diagram shows two right-angled triangles inside a
    square. The perpendicular edges of the larger triangle
    have lengths 15 and 20.
    What is the area of the shaded quadrilateral?

    A 142   B 146   C 150   D 154   E 158

17. Amy, Beth and Claire each has some sweets. Amy gives one third of her sweets to
    Beth. Beth gives one third of all the sweets she now has to Claire. Then Claire
    gives one third of all the sweets she now has to Amy. All the girls end up having
   the same number of sweets.
    Claire begins with 40 sweets. How many sweets does Beth have originally?

    A 20   B 30   C 40   D 50   E 60

18. The arithmetic mean, A, of any two positive numbers x and y is defined to be $A = \frac{1}{2}(x + y)$ and their geometric mean, G, is defined to be $G = \sqrt{xy}$. For two particular values x and y, with $x > y$, the ratio $A : G = 5 : 4$. For these values of x and y, what is the ratio $x : y$?

   A 5 : 4   B 2 : 1   C 5 : 2   D 7 : 2   E 4 : 1

19. The diagram shows a circle of radius 1 touching three sides of a 2 × 4 rectangle. A diagonal of the rectangle intersects the circle at P and Q, as shown. What is the length of the chord PQ?
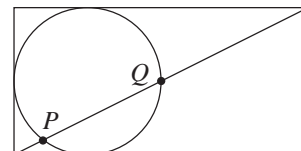
   A $\sqrt{5}$ B $\dfrac{4}{\sqrt{5}}$ C $\sqrt{5} - \dfrac{2}{\sqrt{5}}$ D $\dfrac{5\sqrt{5}}{6}$ E 2

20. The diagram shows a square $PQRS$ with edges of length 1, and four arcs, each of which is a quarter of a circle. Arc TRU has centre P; arc VPW has centre R; arc UV has centre S; and arc WT has centre Q.
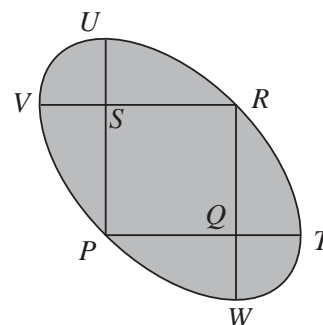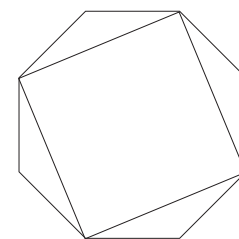
What is the length of the perimeter of the shaded region?

   A 6   B $(2\sqrt{2} - 1)\pi$ C $(\sqrt{2} - \frac{1}{2})\pi$
   D 2π E $(3\sqrt{2} - 2)\pi$

21. How many pairs (x, y) of positive integers satisfy the equation $4^x = y^2 + 15$?

   A 0   B 1   C 2   D 4   E an infinite number

22. The diagram shows a regular octagon and a square formed by drawing four diagonals of the octagon. The edges of the square have length 1.

What is the area of the octagon?

   A $\dfrac{\sqrt{6}}{2}$ B $\dfrac{4}{3}$ C $\dfrac{7}{5}$ D $\sqrt{2}$ E $\dfrac{3}{2}$

23. The parabola with equation $y = x^2$ is reflected in the line with equation $y = x + 2$. Which of the following is the equation of the reflected parabola?

   A $x = y^2 + 4y + 2$ B $x = y^2 + 4y - 2$ C $x = y^2 - 4y + 2$
             D $x = y^2 - 4y - 2$ E $x = y^2 + 2$

24. There is a set of straight lines in a plane such that each line intersects exactly ten others. Which of the following could not be the number of lines in that set?

   A 11   B 12   C 15   D 16   E 20

25. The diagram shows a regular nonagon N. Moving clockwise around N, at each vertex a line segment is drawn perpendicular to the preceding edge. This produces a smaller nonagon S, shown shaded.
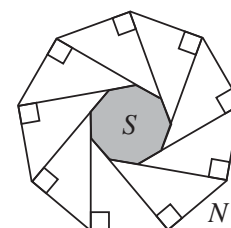
What fraction of the area of N is the area of S?

   A $\dfrac{1 - \cos 40°}{1 + \cos 40°}$ B $\dfrac{\cos 40°}{1 + \cos 40°}$ C $\dfrac{\sin 40°}{1 + \sin 40°}$ D $\dfrac{1 - \sin 40°}{1 + \sin 40°}$ E $\dfrac{1}{9}$